CLASS XII
SAMPLE PAPER-041
MATHEMATICS

Time – 3.00 Hrs.

M.M. – 100

General Instructions:

01. All questions are compulsory.

02. This question paper contains 29 questions.

03. Questions 1 to 4 in **Section A** are very short-answer type questions carrying 1 marks each.

04. Questions 5 to 12 in **Section B** are short-answer type questions carrying 2 marks each.

05. Questions 13 to 23 in **Section C** are long-answer **I** type questions carrying 4 marks each.

06. Questions 24 to 29 in **Section D** are long-answer **II** type questions carrying 6 marks each.

07. Use of calculator is not permitted, you may ask for logarithmic and statistical tables, if required.

Section A

Questions: - 1 to 4 carry 1 marks each.

Q.01 Show that the function $f: R \to R$ given by $f(x) = x^3 + x$ is a bijective.

Q.02 Solve for x : $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}(x), x > 0,$

Q.03 If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

Q.04 for what value of *a*, $\begin{bmatrix} 2a & -1 \\ -8 & 3 \end{bmatrix}$ is a singular matrix ?

Section B

Questions: - 5 to 12 carry 2 marks each.

Q.05Write the value of $\int e^{4\log x}(x^5)\,dx.$

Q.06 If $\vec{a} = 5\hat{\imath} - 4\hat{\jmath} + \hat{k}$, $\hat{b} = -4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\hat{c} = \hat{\imath} - 2\hat{\jmath} - 7\hat{k}$, then find $\hat{c}.(\hat{a} \times \hat{b})$.

Q.07 Find the value of λ such that the line $\frac{x-2}{12} = \frac{y-1}{\lambda} = \frac{z-3}{-8}$ is perpendicular to the plane 3x – y – 2z = 7.

Q.08 If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a})$ = 24, then write the value of $|\vec{x}|$.

Q.09 Let A = $N \times N$ *and* $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c,\ b + d)$, show that $*$ is commutative and associative, Find the identity elements for $*$ on A, if any

Q.10 Solve the equation $\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}] = 0$.

Q.11 Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1 + a^2 + b^2 + c^2)$$

Q.12 For what values of a and b, the function f defined as :

$$f(x) = \begin{cases} 3ax + b, & if < 1 \\ 11, & if\ x = 1 \\ 5ax - 2b, & if\ x > 1 \end{cases} \quad is\ continuous\ at\ x = 1.$$

Section C

Questions: - 13 to 23 carry 4 marks each.

Q.13 If $x^y + y^x = a^b$, $find\ \frac{dy}{dx}$.

OR

If x = a (cost + sint) and y = b (sint – tcos t), find $\frac{d^2y}{dx^2}$.

Q.14 Find the intervals in which the function given by

$$f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi \text{ is}$$

(a) Increasing , *(b)* decreasing

OR

For the curve y = $4x^3$ - $2x^5$, $find$ all points at which the tangent passes through origin.

Q.15 Evaluate: $\int_0^{2\pi} \frac{dx}{1+e^{\sin x}}$.

OR

Evaluate the integral $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$.

Q.16 Form the differential equation of the family of circles in the second quadrant and Touching the coordinate axes.

Q.17 Solve the differential equation :

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1, x \neq 0;$$ when x = 0, y = 1,

Q.18 Let $\vec{a} = \hat{\imath} - \hat{\jmath}$, $\vec{b} = 3\hat{\jmath} - \hat{k}$ and $\vec{c} = 7\hat{\imath} - \hat{k}$. Find the vector $\vec{d}$ which is perpendicular

To both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}$ = 1.

Q.19 Find the distance from the point (3, 4, 5) to the point, where the line

$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane x + y + z = 2.

Q.20 A and B throw a die alternately till one of

Their respective probabilities of winning if A starts the game.

OR

Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls. Find the probability distribution of number of green balls drawn.

Q.21- Using vector method, prove that in a triangle ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Q.22- Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.

Q.23- There are three coins. One is a two headed coin (having heads on both faces). another is a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads. what is the probability that it was the two-headed coin ?

Section D

Questions: - 24 to 29 carry 6 marks each

Q.24 Show that the semi- vertical angle of right circular cone of given surface area and

Maximum volume is $\sin^{-1}\frac{1}{3}$.

OR

P. T. O.

A letter is known to have come either from LONDON or CLIFTON. On the envelop just two consecutive letter ON are visible. What is the probability that letter has come from (1) LONDON ? (ii) CLIFTON ?

Q.25 Evaluate $\int_1^3 (x^2 - x + 1)dx$ using integral as the limit of sums.

Q.26 Find the area lying above x- axis and included between the circles $x^2 + y^2 = 8x$ and the Parabola $y^2 = 4x$.

Q.27 Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured

Parallel to the plane 4x + 12y - 3z + 1 = 0.

Q.28 An urn contains five balls. Two balls are drawn and are found to be white. What is the

Probability that all the balls are white?

Q.29 A small firm manufactures gold rings and chains. The total number of rings and chains Manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum numbers of hours available per day is 16. If a profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should be Manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it Graphically.